



A comparative performance analysis of irreversible Carnot heat engines under maximum power density and maximum power conditions

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Abstract

This paper reports the finite time thermodynamic optimization based on the maximum power density criterion for an irreversible Carnot heat engine model which includes three types of irreversibilities: finite rate heat transfer, heat leakage and internal irreversibility. The obtained results are compared with those results obtained by using the maximum power criterion. The design parameters under the optimal conditions have been derived analytically and the effects of the irreversibilities on the general and optimal performances are investigated. The results showed that the design parameters at maximum power density lead to smaller and more efficient heat engines. It is also seen that the irreversibilities have a greater influence on the performances at maximum power density conditions with respect to the ones at maximum power conditions. Also in this study, the optimal conductance allocation parameter is investigated at both maximum power density and maximum power conditions by assuming a constrained total thermal conductance in the case when there is no heat leakage. The relation between the conductance allocation parameter and the thermal efficiencies at maximum power density and maximum power is also investigated. The obtained results generalize the result of previous studies on this subject and provide guidance to optimal design in terms of power, thermal efficiency and engine sizes for real heat engines. © 1999 Elsevier Science Ltd. All rights reserved.

Keywords: Finite time thermodynamics; Maximum power density; Irreversible Carnot heat engines; heat leak; Optimal performance

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Nomenclature

k	specific heat ratio
m	mass of working fluid
$\dot{m}$	mass flow rate
P	pressure
$\dot{Q}$	rate of heat transfer
$\dot{Q}_{\text{HT}}$	total heat rate transferred from hot reservoir
$\dot{Q}_{\text{LT}}$	total heat rate transferred to cold reservoir
R	ideal gas constant
$R_{\Delta S}$	internal irreversibility parameter
S	entropy
T	temperature
V_4	maximum volume in cycle
$\dot{W}$	power generated from heat engine
x	conductance allocation parameter (α/λ)
α, β, γ	thermal conductance of heat source, heat sink and heat leakage sides, respectively
η	thermal efficiency
λ	overall thermal conductance when no heat leakage ($\alpha + \beta$)
τ	ratio of heat-sink temperature to heat-source temperature

Subscripts

C	Carnot
d	density
H	heat source
L	heat sink
LK	heat leakage
max	maximum
min	minimum
mp	maximum power conditions
opt	optimum
X	warm working fluid
Y	cold working fluid

Superscripts

*	maximum power density conditions
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1. Introduction

Power optimization studies of heat engines using finite time thermodynamics analysis were started by Chambadal [1] and Novikov [2] in 1957 and 1958 and were continued by Curzon

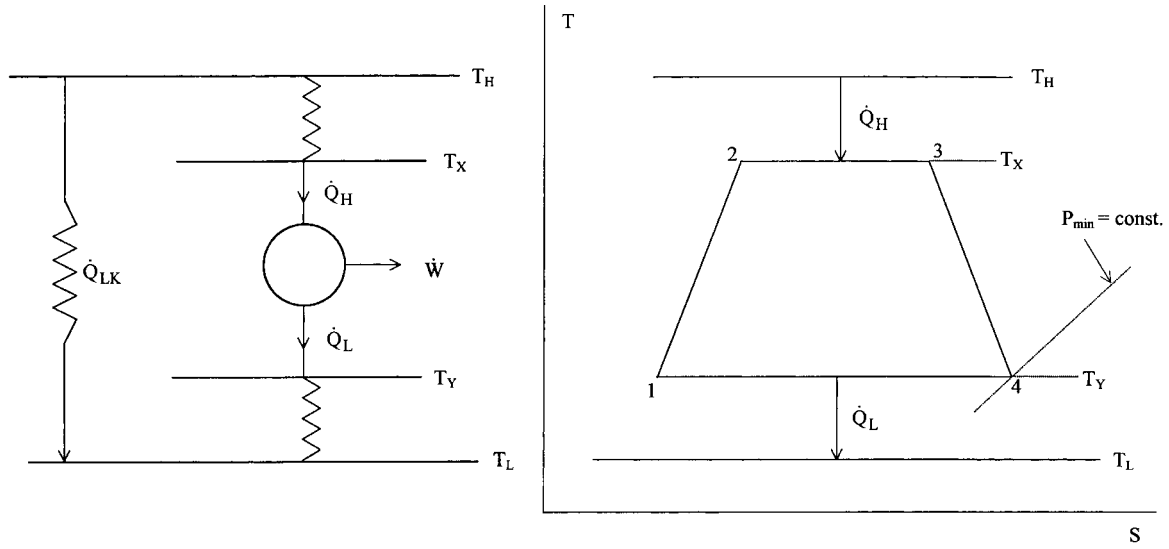
and Ahlborn [3] in 1975. They extended the reversible Carnot cycle to the endoreversible cycle by taking the irreversibility of finite-time heat transfer into account. By introducing finite time processes, a new field: ‘finite time thermodynamics’ was born, and thus, more realistic limits have been adapted for real heat engine performances. During the last decade, many optimization studies for heat engines based on endoreversible and irreversible models have been performed by considering finite time and finite size constraints under various heat transfer modes, mainly linear and non-linear ones. The reader who is interested in these optimization works could refer to a literature survey written by Bejan [4]. Usually, in these studies, the power output, thermal efficiency, entropy generation and the ecological benefit were chosen for optimization criteria. However, the performance analyses based on the above optimization criteria do not take the effect of engine sizes related to the investment cost into account. In order to include the effects of engine size in the performance analysis, Sahin et al. [5] introduced a new optimization criterion called the maximum power density (MPD) analysis. Using the MPD criterion, they investigated optimal performance conditions for reversible [5] and irreversible [6] non-regenerative Joule–Brayton heat engines. In their study, they maximized the power density (the ratio of power to the maximum specific volume in the cycle) and found design parameters at MPD conditions, which lead to smaller and more efficient Joule–Brayton engines than those engines working at maximum power (MP) conditions. Erbay et al. [7], Erbay and Yavuz [8], Chen et al. [9] and Medina et al. [10] applied the MPD technique to the Ericsson, Stirling, Atkinson and regenerative Joule–Brayton engines, respectively. In their analyses, they discussed the advantages of the MPD performance conditions in comparison to the MP conditions in terms of thermal efficiency and engine sizes. Sahin et al. [11] applied the MPD technique to the endoreversible Carnot heat engine which can be considered as a theoretical comparison standard for all real heat engines in finite time thermodynamics and thus generalized the endoreversible MPD analyses results. They also demonstrated that the thermal efficiency at MPD conditions varies between $1 - \sqrt{\tau}$ and $[1 - 2\tau/(1 + \tau)]$ for $0 \leq x \leq 1$ where τ is the ratio of heat sink to heat source temperature and x is the conductance allocation parameter.

In this paper, the MPD performance analysis performed by Sahin et al. [11] for the endoreversible Carnot heat engine model is extended to an irreversible Carnot heat engine model by considering internal irreversibility and heat leakage effects. In this context, the optimal performance and design parameters under MPD conditions are investigated. The obtained results are comparatively discussed with respect to the results obtained by using the MP performance criterion.

2. The theoretical model

The considered irreversible Carnot heat engine model which includes heat leakage, finite time heat transfer and internal irreversibilities and its T – S diagram are shown in Fig. 1. The rate of heat leakage $\dot{Q}_{LK}$ from the hot reservoir at temperature T_H to the cold reservoir at temperature T_L with thermal conductance γ is given by

$$\dot{Q}_{LK} = \gamma(T_H - T_L). \quad (1)$$

Fig. 1. Irreversible Carnot heat engine model and its T - S diagram.

The heat rate $\dot{Q}_H$ from the hot reservoir at temperature T_H to the heat engine with thermal conductance α and the heat rate $\dot{Q}_L$ from the heat engine to the cold reservoir at temperature T_L with thermal conductance β , are respectively,

$$\dot{Q}_H = \alpha(T_H - T_X), \quad (2)$$

$$\dot{Q}_L = \beta(T_Y - T_L). \quad (3)$$

Then, the total heat rate $\dot{Q}_{HT}$ transferred from the hot reservoir is

$$\dot{Q}_{HT} = \dot{Q}_H + \dot{Q}_{LK} = \alpha(T_H - T_X) + \gamma(T_H - T_L) \quad (4)$$

and the total heat rate $\dot{Q}_{LT}$ transferred to the cold reservoir is

$$\dot{Q}_{LT} = \dot{Q}_L + \dot{Q}_{LK} = \beta(T_Y - T_L) + \gamma(T_H - T_L), \quad (5)$$

where T_X and T_Y are the warm and cold working fluid temperatures, respectively. The power produced by the engine according to the first law is

$$\dot{W} = \dot{Q}_{HT} - \dot{Q}_{LT} = \dot{Q}_H - \dot{Q}_L = \alpha(T_H - T_X) - \beta(T_Y - T_L). \quad (6)$$

The power density defined as the ratio of power to the maximum volume in the cycle then takes the form

$$\dot{W}_d = [\alpha(T_H - T_X) - \beta(T_Y - T_L)]/V_4. \quad (7)$$

Formally, the second law for an irreversible cycle requires that

$$\oint \frac{\delta \dot{Q}}{T} = \frac{\dot{Q}_H}{T_X} - \frac{\dot{Q}_L}{T_Y} < 0. \quad (8)$$

One can rewrite the inequality in Eq. (8) as

$$\frac{\dot{Q}_H}{T_X} - R_{\Delta S} \frac{\dot{Q}_L}{T_Y} = 0 \quad \text{with} \quad 0 < R_{\Delta S} < 1 \quad (9)$$

where the internal irreversibility parameter $R_{\Delta S}$ is defined as

$$R_{\Delta S} = \frac{S_3 - S_2}{S_4 - S_1} \quad (10)$$

which characterizes the degree of internal irreversibility. By substituting Eqs. (2) and (3) in Eq. (9), we have

$$\alpha(T_H - T_X)/T_X - R_{\Delta S}\beta(T_Y - T_L)/T_Y = 0. \quad (11)$$

Assuming an ideal gas, the maximum volume in the cycle V_4 can be written as

$$V_4 = mRT_Y/P_{\min}, \quad (12)$$

where m is the mass of the working fluid and R is the ideal gas constant. In the analysis, the minimum pressure ($P_{\min}$) in the cycle is taken to be constant. The power density then becomes

$$\dot{W}_d = (P_{\min}/mR)[\alpha(T_H - T_X) - \beta(T_Y - T_L)]/T_Y. \quad (13)$$

One can maximize the power density given in Eq. (13) with respect to T_X and T_Y by using Eq. (11). The results are

$$T_X^* = T_H \sqrt{(\alpha + \beta\tau)/(\alpha + \beta R_{\Delta S})}, \quad (14)$$

$$T_Y^* = \beta R_{\Delta S} T_L / \left[\alpha + \beta R_{\Delta S} - \alpha \sqrt{(\alpha + \beta R_{\Delta S})/(\alpha + \beta\tau)} \right], \quad (15)$$

where $\tau = T_L/T_H$. Substituting Eqs. (14) and (15) into Eq. (13), the MPD can be found as

$$\dot{W}_{d_{\max}} = (\alpha P_{\min}/mR) \left[\sqrt{\alpha + \beta R_{\Delta S}} - \sqrt{\alpha + \beta\tau} \right]^2 / \beta\tau R_{\Delta S}. \quad (16)$$

The thermal efficiency,

$$\eta = 1 - \dot{Q}_{LT}/\dot{Q}_{HT} \quad (17)$$

can be written, after some manipulations as

$$\eta = [1 - T_Y/(R_{\Delta S} T_X)] / \left\{ 1 + [\gamma(1 - \tau)(\alpha + \beta R_{\Delta S}) T_Y/T_X] / [\alpha \beta R_{\Delta S} (T_Y/T_X - \tau)] \right\}. \quad (18)$$

Using Eqs. (14) and (15) in Eq. (18), the efficiency at MPD becomes

$$\eta^* = \eta_{\gamma=0}^* / \left\{ 1 + \gamma(1 - \tau) / \left[\alpha \left(1 - \sqrt{(\alpha + \beta\tau)/(\alpha + \beta R_{\Delta S})} \right) \right] \right\} \quad (19)$$

where $\eta_{\gamma=0}^*$ is the efficiency at MPD when there is no heat leakage, and it can be obtained as

$$\eta_{\gamma=0}^* = 1 - \beta\tau / \left(\sqrt{(\alpha + \beta\tau)(\alpha + \beta R_{\Delta S})} - \alpha \right). \quad (20)$$

Similarly, one can maximize the power given in Eq. (6) with respect to T_X and T_Y using the condition given in Eq. (11) and obtain

$$T_{X_{mp}} = \left(\alpha T_H + \beta \sqrt{R_{\Delta S} T_L T_H} \right) / (\alpha + \beta R_{\Delta S}) \quad (21)$$

$$T_{Y_{mp}} = \left(\beta R T_L + \alpha \sqrt{R_{\Delta S} T_L T_H} \right) / (\alpha + \beta R_{\Delta S}). \quad (22)$$

Substituting Eqs. (21) and (22) into Eq. (6), the maximum power can be found as

$$\dot{W}_{max} = \alpha \beta T_H \left(\sqrt{R_{\Delta S}} - \sqrt{\tau} \right)^2 / (\alpha + \beta R_{\Delta S}) \quad (23)$$

and the thermal efficiency at maximum power is

$$\eta_{mp} = \frac{1 - \sqrt{\tau/R_{\Delta S}}}{1 + [\gamma(1 - \tau)(\alpha + \beta R_{\Delta S})] / [\alpha \beta R_{\Delta S} (1 - \sqrt{\tau/R_{\Delta S}})]}. \quad (24)$$

3. Results and discussion

The variations of the normalized power density and the normalized power with respect to thermal efficiency are shown for the zero heat leakage case ($\gamma = 0$) in Fig. 2(a) and (b), and for the heat leakage case ($\gamma > 0$) in Fig. 3(a) and (b), respectively. The following comments can be summarized from the observation of these figures: (i) for the chosen values of the parameters, although the maximum thermal efficiencies (η_{max}) are the same, the thermal efficiency at MPD (η^*) is greater than the thermal efficiency at MP conditions (η_{mp}), i.e., $\eta_{max} > \eta^* > \eta_{mp}$; (ii) an increase in τ (the ratio of heat sink to heat source temperatures) and the effect of heat leakage cause the thermal efficiencies η_{max} , η^* and η_{mp} to get closer to each other, that is the thermal efficiency advantage of the MPD conditions with respect to the MP conditions ($\Delta\eta = \eta^* - \eta_{mp}$) decreases; (iii) the heat leakage effect causes the loop shaped performance curves and also causes decreased values of the optimal thermal efficiencies, e.g. for $\tau = 0.1$ and $R_{\Delta S} = 0.8$ values when there is no heat leakage ($\gamma = 0$), $\eta_{max} = 0.88$, $\eta^* = 0.75$ and $\eta_{mp} = 0.64$ and when there is heat leakage ($\gamma/\alpha = 0.05$), $\eta_{max} = 0.62$, $\eta^* = 0.60$ and $\eta_{mp} = 0.55$. The above evaluations, drawn from observation of Figs. 2 and 3 can be seen more clearly from Fig. 4(a). The variations of the thermal efficiencies at MPD and MP conditions (η^* and η_{mp}) and the variations of the dimensionless powers at MPD and MP conditions [$\dot{W}^* = \dot{W}_{d_{max}} v_4^* / \alpha T_H$, $\dot{W}_{mp} = \dot{W}_{max} / \alpha T_H$] with respect to τ can be seen from Fig. 4(a) and (b), respectively. In Fig. 4(a), the Carnot efficiency ($\eta_C = 1 - \tau$) is also given for comparison reasons. As can be seen from Fig.

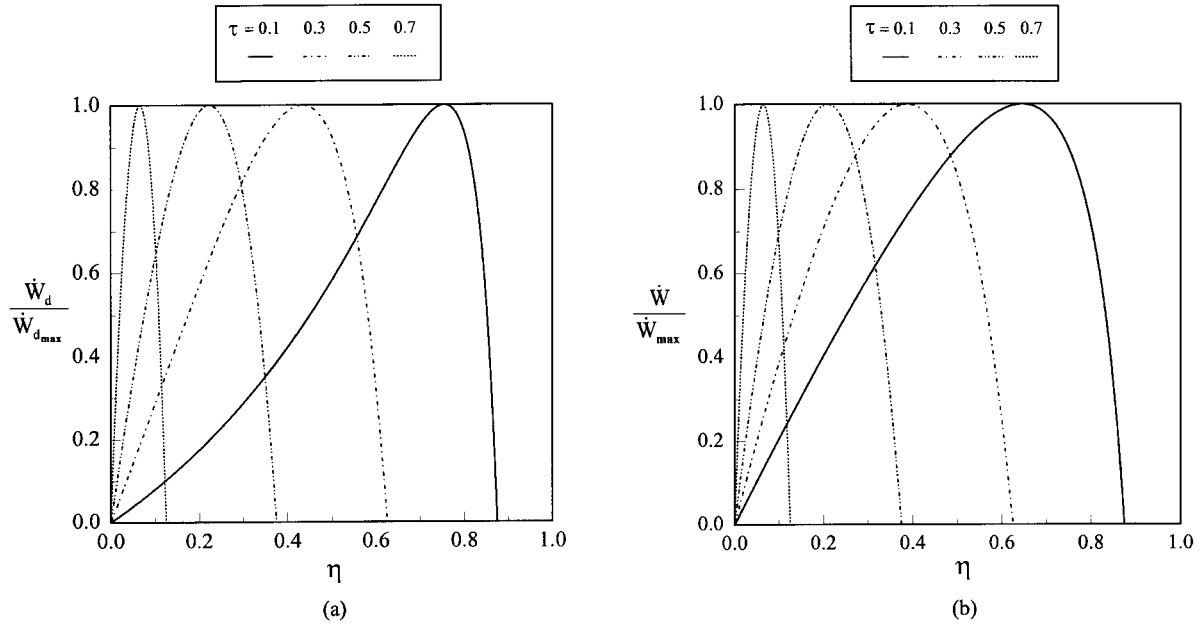


Fig. 2. Variations of the normalized power density (a) and the normalized power (b) with respect to the thermal efficiency ($\alpha = \beta$, $\gamma = 0$, $R_{DS} = 0.8$).

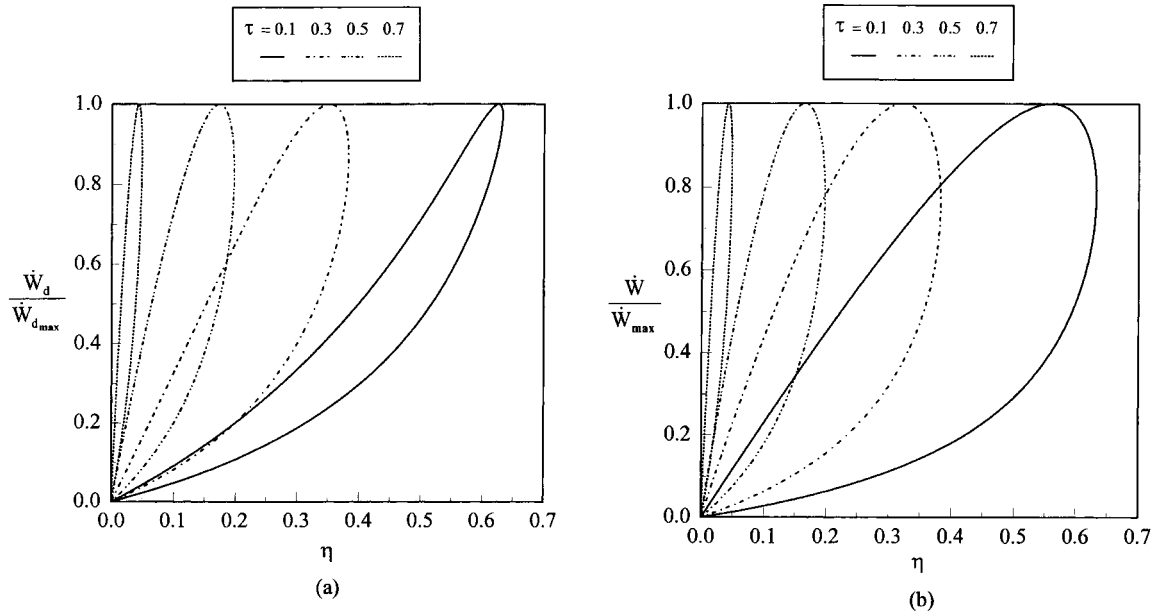


Fig. 3. Variations of the normalized power density (a) and the normalized power (b) with respect to the thermal efficiency ($\alpha = \beta$, $\gamma/\alpha = 0.05$, $R_{DS} = 0.8$).

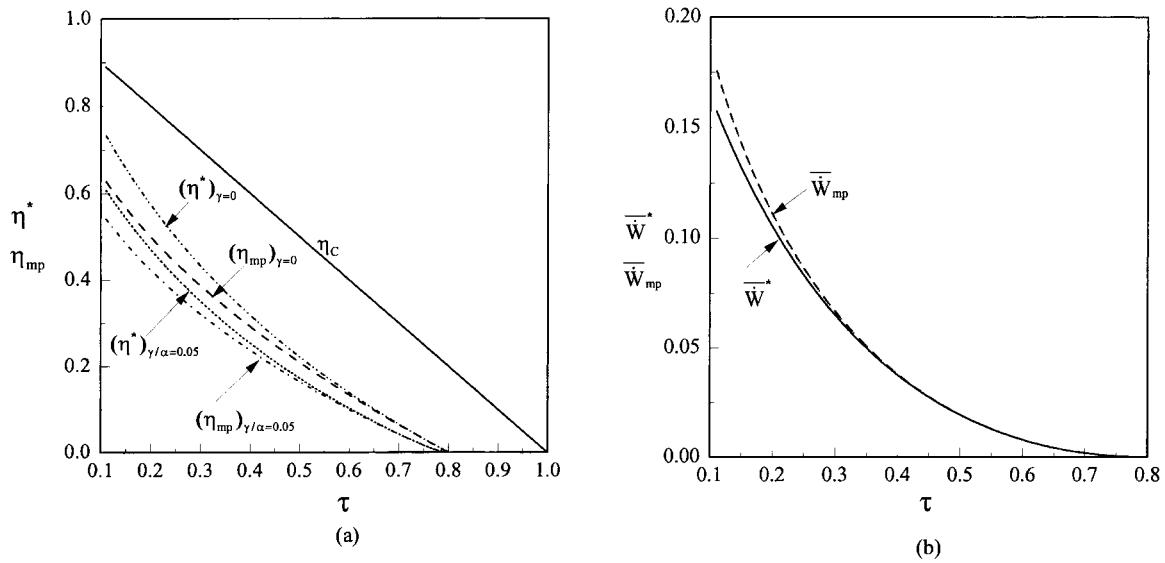


Fig. 4. Variations of the thermal efficiencies (a) and dimensionless powers (b) at MPD and MP conditions with respect to τ , ($\alpha = \beta$, $R_{\Delta S} = 0.8$).

4(a) and (b), η^* is greater than η_{mp} and $\dot{W}^*$ is lower than $\dot{W}_{max}$, that is if the design parameters are selected at MPD conditions instead of MP conditions the thermal efficiency increases as much as $\Delta\eta = \eta^* - \eta_{mp}$, but the power reduces by $\Delta\dot{W} = \dot{W}_{max} - \dot{W}^*$. On the other hand, as τ increases, η^* and η_{mp} , and $\dot{W}^*$ and $\dot{W}_{max}$ decrease and get closer to each other, respectively and they become zero when $\tau = R_{\Delta S}$. In order to obtain positive performance in terms of thermal efficiency and power for both the MPD and MP conditions, it is necessary that $\tau < R_{\Delta S}$. This case can be seen by evaluating Eqs. (16), (19), (23), (24). The variations of the dimensionless power density [$\bar{W}_d = \dot{W}_d/(\alpha P_{min}/mR)$] and the dimensionless power [$\bar{W} = \dot{W}/(\alpha T_H)$] with respect to thermal efficiency are shown for various internal irreversibility parameters ($R_{\Delta S}$) in Fig. 5(a) and (b), and for various heat leakage values in Fig. 6(a) and (b), respectively. From these figures, one can observe the effects of the internal irreversibility and heat leakage on the global and optimal performances. As the internal irreversibility and the heat leakage increase, that is, as $R_{\Delta S}$ decreases and γ/α increases, the global and the optimal performances gradually decrease and the values of the efficiencies η_{max} , η^* and η_{mp} get very close to each other. In Fig. 7(a) and (b), the effects of the internal irreversibility and heat leakage on the thermal efficiencies at MPD and MP conditions can be seen more clearly. As can be seen from the figures, η^* is greater than η_{mp} for all values of $R_{\Delta S}$ and γ/α . However, the advantage of η^* with respect to η_{mp} ($\Delta\eta = \eta^* - \eta_{mp}$) decreases as the internal irreversibility and heat leakage increase. Therefore, the reducing effects of the internal irreversibility and heat leakage on η^* is greater in comparison to η_{mp} . Thus, it is more important to take precautions to reduce internal irreversibility and heat leakage in the design of a heat engine working at MPD conditions with respect to the one working at MP conditions.

In order to determine optimal thermal conductances, which are measures of heat exchanger

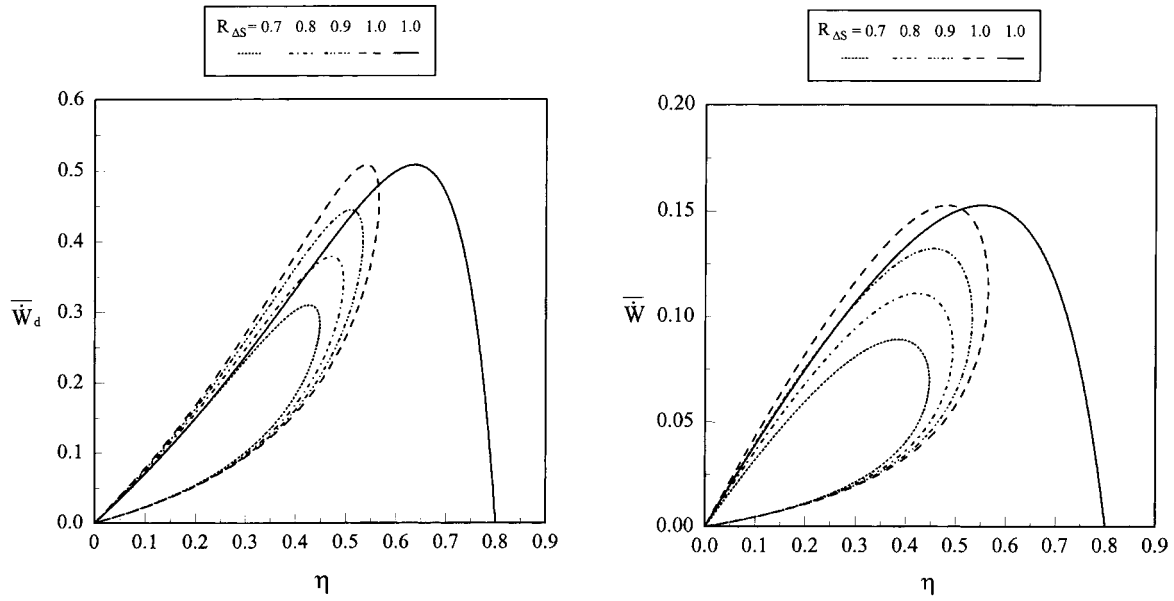


Fig. 5. Variations of the dimensionless power density (a) and the dimensionless power (b) with respect to the thermal efficiency, for various internal irreversibility parameters, ($\tau = 0.2$, $\alpha = \beta$, $\gamma/\alpha = 0.05$, for the solid lines $\gamma = 0$).

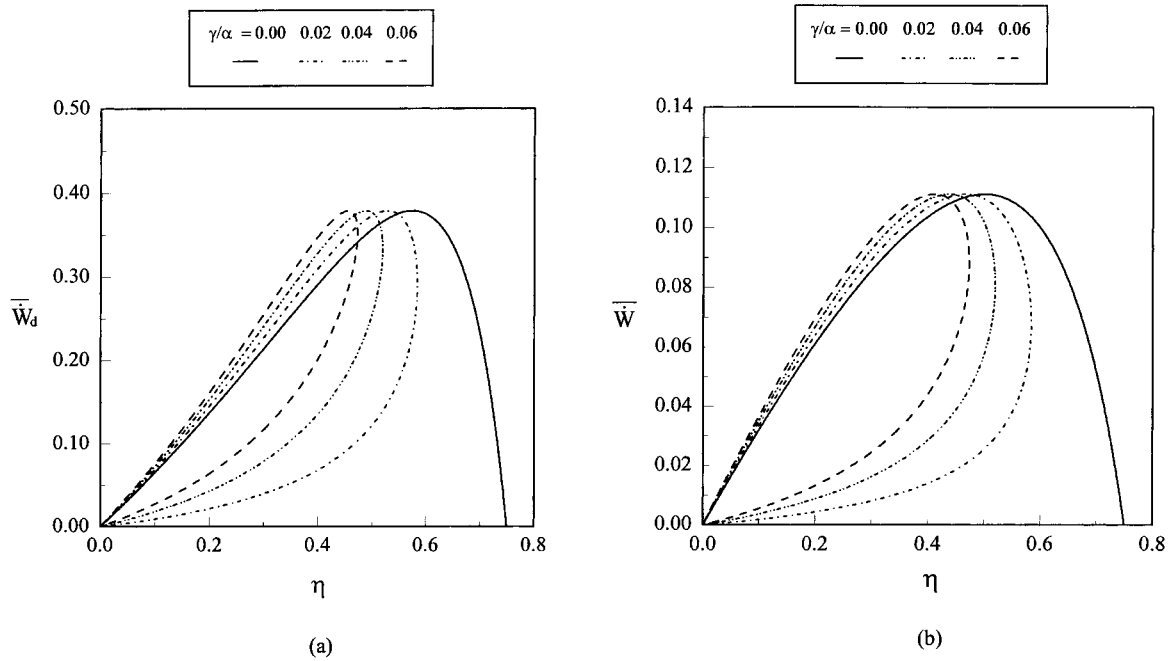


Fig. 6. Variations of the dimensionless power density (a) and the dimensionless power (b) with respect to the thermal efficiency, for various heat leakage values, ($\tau = 0.2$, $\alpha = \beta$, $R_{\Delta S} = 0.8$).

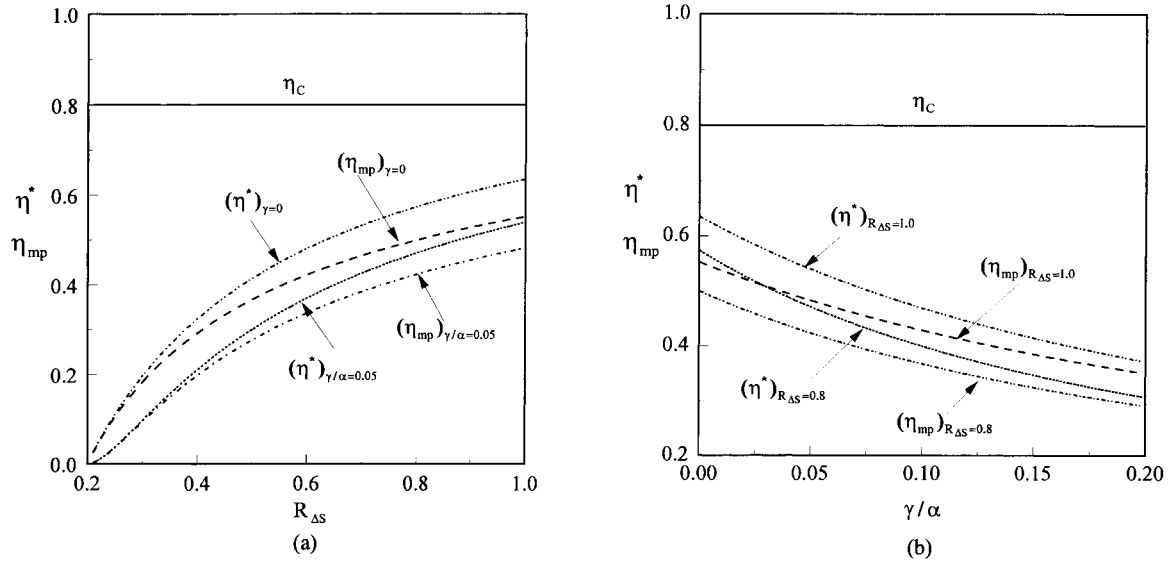


Fig. 7. Effects of the internal irreversibility (a) and heat leakage (b) on the thermal efficiencies at MPD and MP conditions, ($\tau = 0.2$, $\alpha = \beta$).

sizes, we assume that the total thermal conductance is constrained, and there is no heat leakage, i.e

$$\alpha + \beta = \lambda = \text{constant}. \quad (25)$$

Let us define a thermal conductance allocation ratio x for the heat source side as

$$\alpha = x\lambda, \quad (26)$$

and from Eq. (25) for the heat sink side

$$\beta = (1 - x)\lambda. \quad (27)$$

By using these definitions, the maximum power density ($\dot{W}_{d_{\max}}$) given in Eq. (16) becomes,

$$\dot{W}_{d_{\max}} = \frac{\lambda P_{\min}}{mR} \frac{x \left[\sqrt{x + (1 - x)R_{\Delta S}} - \sqrt{x + (1 - x)\tau} \right]^2}{(1 - x)\tau R_{\Delta S}} \quad (28)$$

and the thermal efficiency at MPD given in Eq. (20) becomes,

$$\eta^* = 1 - \frac{(1 - x)\tau}{\sqrt{[x + (1 - x)\tau][x + (1 - x)R_{\Delta S}] - x}}. \quad (29)$$

The effects of the conductance allocation parameter x on dimensionless maximum power density $\dot{W}_{d_{\max}} = \dot{W}_{d_{\max}}/(\lambda P_{\min}/mR)$ are shown in Fig. 8(a) and (b) for various τ and $R_{\Delta S}$ values. As can be seen from the figures, the optimal conductance allocation parameter x which maximizes Eq. (28) does not much affected by τ and $R_{\Delta S}$ and it is approximately $x_{\text{opt}}^* = 0.4$.

For the maximum power conditions, by using Eqs. (26) and (27) in Eq. (23), we have

$$\dot{W}_{\max} = \frac{\lambda x(1-x)[\sqrt{R_{\Delta S}T_H} - \sqrt{T_L}]^2}{x + (1-x)R_{\Delta S}} \quad (30)$$

and the thermal efficiency at MP given in Eq. (24) becomes,

$$\eta_{\text{mp}} = 1 - \sqrt{\tau/R_{\Delta S}}. \quad (31)$$

The optimal value of x which yields the maximum of the maximum power can be found by maximizing Eq. (30) with respect to x . The result of this optimization is

$$(x_{\text{mp}})_{\text{opt}} = \frac{1}{R_{\Delta S}^{-0.5} + 1} \quad (32)$$

which is independent of τ and only depends on $R_{\Delta S}$. The optimal conductance allocation parameter at MPD is approximately constant around 0.4 (not much affected by τ and $R_{\Delta S}$), but the optimal conductance allocation parameter at MP changes with $R_{\Delta S}$. For example, in the endoreversible case ($R_{\Delta S} = 1$), $(x_{\text{mp}})_{\text{opt}} = 0.5$ and it reduces to 0.47 for an irreversible case when $R_{\Delta S} = 0.8$. When we compare the optimal conductance allocation parameters at MPD and MP conditions, we observe that $(x_{\text{mp}})_{\text{opt}} > x_{\text{opt}}^*$ for $R_{\Delta S} > 0.45$. The relations between the optimal thermal efficiencies (η^* and η_{mp}) and the conductance allocation parameter can be seen from Eqs. (29) and (31). As is seen, η_{mp} is independent of x but η^* depends on x , this case can also be seen from Fig. 8(c). This figure and Eq. (29) indicate that, as $x \rightarrow 1$, η^* approaches $1 - 2\tau/(\tau + R_{\Delta S})$ and for $x = 0$, $\eta^* = \eta_{\text{mp}} = 1 - \sqrt{\tau/R_{\Delta S}}$, i.e.

$$1 - \sqrt{\tau/R_{\Delta S}} \leq \eta^* \leq 1 - 2\tau/(\tau + R_{\Delta S}) \text{ for } 0 \leq x \leq 1. \quad (33)$$

This result generalizes the result previously shown by Sahin et al. [11].

The size of a heat engine can be characterized by the maximum volume in the cycle, i.e. V_4 . For example, in reciprocating engines, the engine size is determined by the sum of the clearance and displacement volumes (i.e. the maximum volume in the cycle). For turbomachinery, Sahin et al. [5] discussed that the maximum volume in the cycle characterizes the engine size. The ratio of the maximum volume at MPD to the one at MP can be written as

$$\frac{V_4^*}{(V_4)_{\text{mp}}} = \frac{(1-x)\sqrt{R_{\Delta S}\tau}[x + (1-x)R_{\Delta S}][x + (1-x)\tau]}{[x + (1-x)\sqrt{R_{\Delta S}\tau}]\left\{\sqrt{[x + (1-x)R_{\Delta S}][x + (1-x)\tau]} - x\right\}}. \quad (34)$$

similarly the ratio of the maximum pressures at MPD and MP conditions is

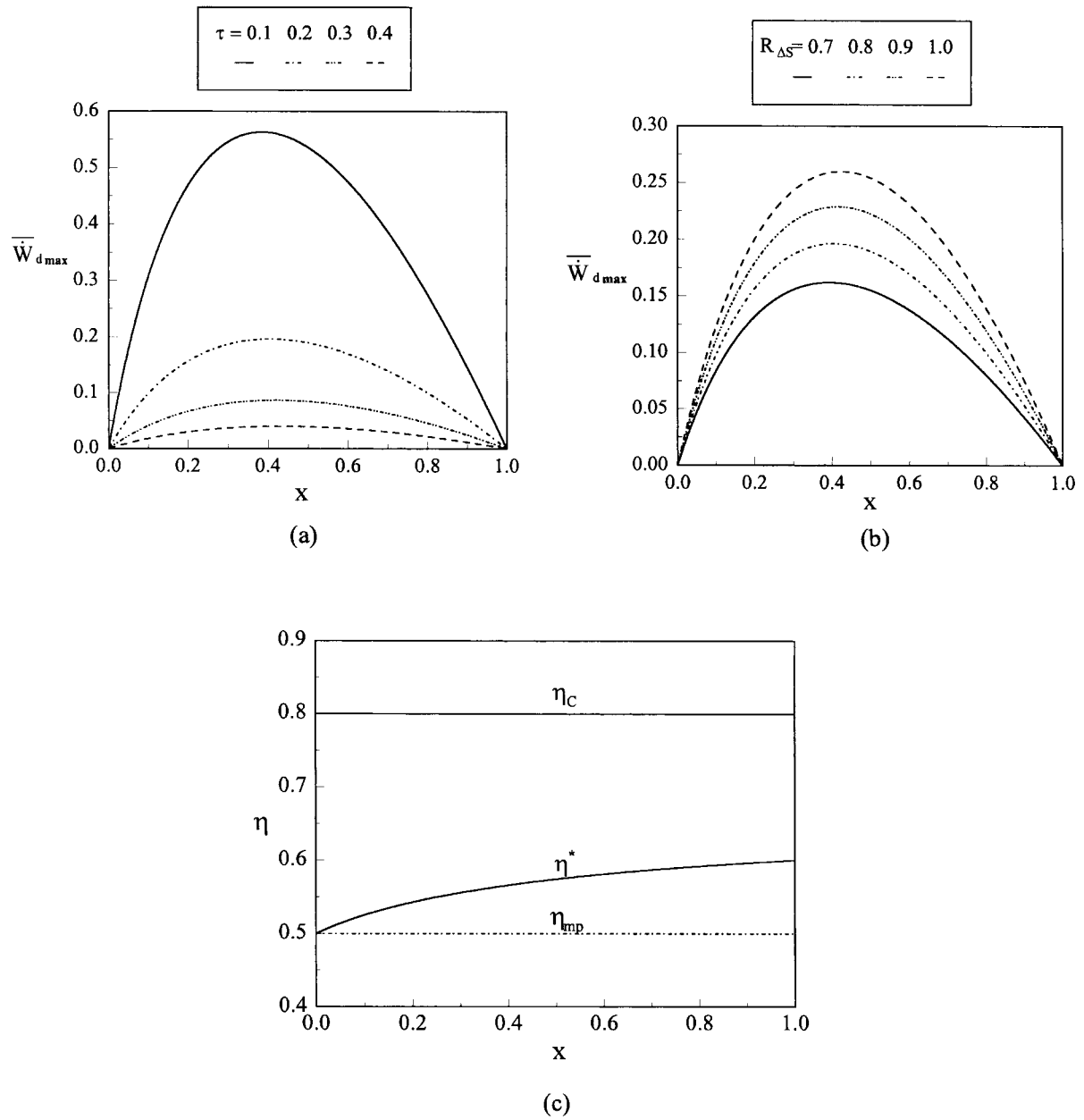


Fig. 8. Effect of the conductance allocation parameter x on dimensionless maximum power density for various τ (a), ($R_{\Delta S} = 0.8$, $\gamma = 0$); and for various $R_{\Delta S}$ values (b), ($\tau = 0.2$, $\gamma = 0$); and the effect of x on thermal efficiencies at MPD and MP (c), ($\tau = 0.2$, $R_{\Delta S} = 0.8$, $\gamma = 0$).

$$\frac{P_{\max}^*}{(P_{\max})_{\text{mp}}} = \exp \left[\frac{x\lambda T_H}{2\dot{m}R} \left(1 + \frac{1}{R_{\Delta S}} \right) \left(\frac{1}{T_X^*} - \frac{1}{T_{X_{\text{mp}}}} \right) \right] \left(\frac{T_Y^* T_{X_{\text{mp}}}}{T_X^* T_{Y_{\text{mp}}}} \right)^{k/1-k}. \quad (35)$$

In the derivation of Eq. (35), it is assumed that $S_2 - S_1 = S_4 - S_3$. Variations of $V_4^*/(V_4)_{\text{mp}}$ and $P_{\max}^*/(P_{\max})_{\text{mp}}$ are plotted with respect to τ for the endoreversible case ($R_{\Delta S} = 1$) and an irreversible case ($R_{\Delta S} = 0.8$) in Fig. 9. As can be seen from the figure, the ratio of the maximum volumes in the cycle is less than unity and the ratio of the maximum pressures in the cycle is greater than unity. As a result, at MPD conditions engine sizes will be smaller but will require higher maximum pressure than the one operating at MP. On the other hand, as internal irreversibility increases, that is, as $R_{\Delta S}$ decreases, the engine size advantage of the MPD conditions decreases. However, its maximum pressure disadvantage gets better.

4. Conclusions

A comparative performance analysis based on MPD and MP criteria has been performed for irreversible Carnot heat engines which include the major irreversibilities existing in real heat engines. The analysis showed that the thermal efficiency at MPD conditions is greater than the one at MP conditions, for an irreversible heat engine. However, the thermal efficiency

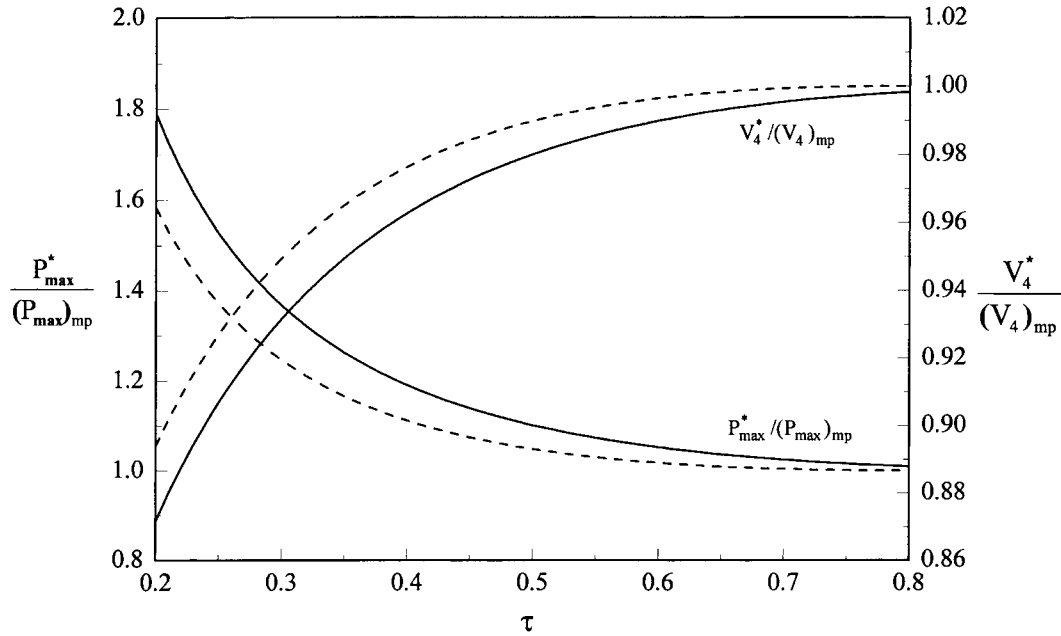


Fig. 9. The variation of the ratio of the maximum volume in the cycle at MPD to that at MP and the variation of the ratio of maximum pressure at MPD to that at MP with respect to τ for endoreversible case ($R_{\Delta S} = 1$, solid lines) and irreversible case ($R_{\Delta S} = 0.8$, dashed lines), ($x = 0.5$, $\gamma = 0$, $T_L = 300$ K, $\lambda/\dot{m}R = 3$).

advantage of the MPD conditions with respect to MP conditions decreases as the internal irreversibility and heat leakage increase. This result indicates that the reducing effect of the internal irreversibility and heat leakage on the thermal efficiency at MPD conditions is greater in comparison to the one at MP conditions. Thus, it is more important to take precautions to reduce internal irreversibility and heat leakage in the design of a heat engine working at MPD conditions with respect to the one working at MP conditions. It is found that the optimal conductance allocation parameter at MPD conditions (x_{opt}^*) is approximately constant around 0.4, and it is not much affected by τ and $R_{\Delta S}$ (Fig. 8(a) and (b)) but the optimal conductance-allocation parameter at MP conditions [$(x_{\text{mp}})_{\text{opt}}$] changes with $R_{\Delta S}$ (see Eq. (32)). When we compare the optimal conductance-allocation parameters at MPD and MP conditions, we observe that $(x_{\text{mp}})_{\text{opt}} > x_{\text{opt}}^*$ for $R_{\Delta S} > 0.45$. When there is no heat leakage, the thermal efficiency at MP conditions is independent of the conductance-allocation parameter (see Eq. (31)) but the thermal efficiency at MPD conditions depends on the conductance-allocation parameter (Eq. (29)). Eq. (29) indicates that the thermal efficiency at MPD conditions varies between $1 - \sqrt{\tau/R_{\Delta S}}$ and $1 - 2\tau/(\tau + R_{\Delta S})$ for $0 \leq x \leq 1$. Furthermore, it is shown that engine sizes designed at MPD conditions would be smaller but require higher maximum pressure than those operating at MP conditions. The results presented in this analysis generalize the results of the previous studies on this subject and may provide guidelines for determination of the optimal design and operating conditions of real heat engines.

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